Inequality

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Let $x, y, z > 1$, prove that:

$$\sqrt{(x^2-1)} + \sqrt{(y^2-1)} + \sqrt{(z^2-1)} \le (xy+yz+zx)/2.$$

Generalize!

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We will prove that for any real $x_1, x_2, \ldots, x_n > 1$, where $n \ge 2$, holds inequality

(**1**) $\quad \sum_{k=1}^{n} \sqrt{x_k^2 - 1} \le \frac{1}{2} \sum_{k=1}^{n} x_k x_{k+1}$ (here $x_{n+1} = x_1$).

Let $a_k := \sqrt{x_k^2 - 1}, k = 1, 2, \ldots, n$. Then $x_k = \sqrt{a_k^2 + 1}, k = 1, 2, \ldots, n$ and (**1**)$\Leftrightarrow$

(**2**) $\quad \sum_{k=1}^{n} a_k \le \frac{1}{2} \sum_{k=1}^{n} \sqrt{a_k^2 + 1} \cdot \sqrt{a_{k+1}^2 + 1}$, where $a_k > 0, k = 1, 2, \ldots, n$ and $a_{n+1} = a_1$.

Using Cauchy Inequality we obtain

$$\frac{1}{2} \sum_{k=1}^{n} \sqrt{a_k^2 + 1} \cdot \sqrt{a_{k+1}^2 + 1} = \frac{1}{2} \sum_{k=1}^{n} \sqrt{a_k^2 + 1} \cdot \sqrt{1 + a_{k+1}^2} \ge$$

$$\frac{1}{2} \sum_{k=1}^{n} (a_k \cdot 1 + 1 \cdot a_{k+1}) = \frac{1}{2} \sum_{k=1}^{n} (a_k + a_{k+1}) = \sum_{k=1}^{n} a_k.$$